Mind the Gap

reflections on benchmarking in the global era

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"Challenges in Benchmarking Optimization Heuristics"

Consensus on Benchmarking Compilation?

The **human factor plays a crucial role in such a process**. Formulation of a test-suite may involve three types of scholars: theoreticians, algorithms' designers, and practitioners:

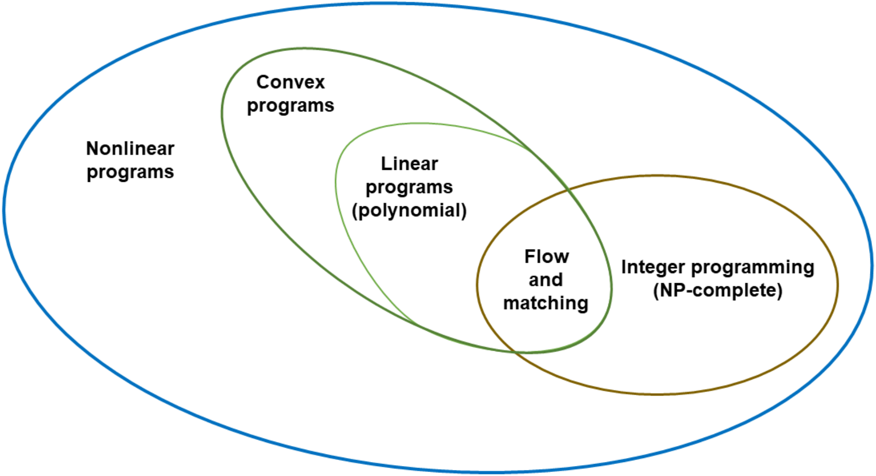
(i) **theoreticians** naturally favor analyzable functions

(ii) **algorithms' engineers** may prefer families of functions that are successfully treated by their designs

(iii) **practitioners** may have the best insights into which functions most accurately represent real-world problems (thus having their biased preferences)

A proper balance should be made amongst those three parties to effectively compile a test-suite meaningful to a broad audience.[1]

Yet Another Perspective: Operations Research as the Optimization Complement[2]



Opportunities: OR Gaps

Could we perhaps make a difference by actually addressing the weak spots of OR as our benchmarks?

Algorithms-wise, hybrids do make a difference while outperforming white-box solvers – see, e.g., MATHEURISTICS and Blum+Raidl.[3]

Existing opportunity:
Aim for compiling an "OpenEC" platform, with a unified interface, for optimizing test-functions that are beyond the reach of OR.

Continuous Opportunity (SO): The Quantum Sphere[4]

Given a control phase $\phi(\Omega)$ and a dispersion term $k(\Omega)$, the spectral intensity $I_2(\Omega)$ is formulated by means of the *spectral* terms:

$$\begin{aligned} E_1(\Omega) &= A(\Omega)\exp i\left[\phi(\Omega)+k(\Omega)\right] \\ E_2(\Omega) &= E_1(\Omega) * E_1(\Omega) = \int_{-\infty}^{\infty} E_1(\Omega')\cdot E_1(\Omega-\Omega')\mathrm{d}\Omega' \\ I_2(\Omega) &= \left|E_2(\Omega)\right|^2. \end{aligned} \tag{1}$$

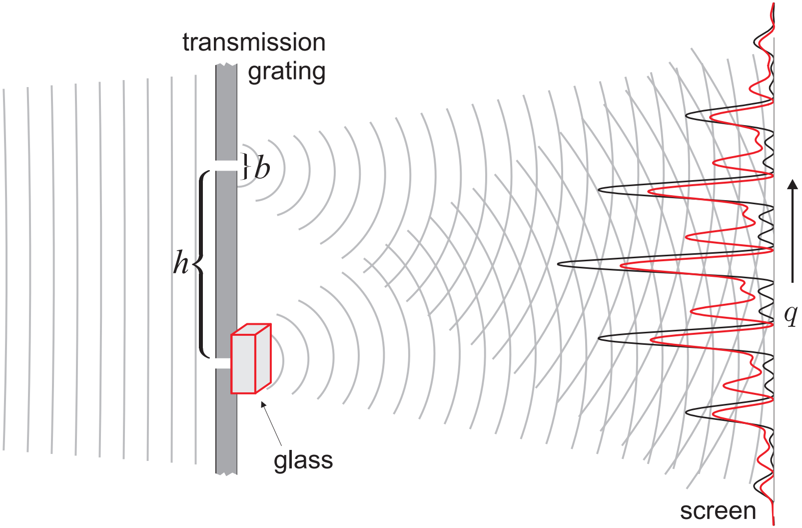
The standard Quantum Sphere is defined as follows:

$$\boxed{QS\left(\phi,k\right) = \int_{-\infty}^{\infty} I_2(\Omega)\mathrm{d}\Omega \longrightarrow \max}\ . \tag{2}$$

When $k=0$, QS is maximized by any phase function linear in the frequency Ω: $\text{argmax}_{\phi(\Omega)}\left\{QS\left(\phi,k=0\right)\right\} \equiv a\Omega+b \quad \forall a,b$ (and in particular by a constant phase, $a=0$, $\forall b$).

$\Longrightarrow$ The decision variables ϕ are meant to compensate for the function $k(\Omega)$ over the **periodic domain** $[0,2\pi]^D$.

Continuous Opportunity (MO): Diffraction Grating[5]



DG Formulation

Introducing a basic set of optical test-problems for Pareto optimization, scalable in dimension and subject to a collection of defining parameters for setting the Pareto front's curvature.

Given an optical setup of n slits, and given phases $\vec{\varphi} \in [0, 2\pi]^n$, the intensity on a screen point positioned at q reads:

$$I_{DG}(q, \vec{\varphi}) = \frac{1}{n^2}\text{sinc}^2\left(\frac{qb}{2}\right) \cdot \left|\sum_{k=0}^{n-1} \exp(iqhk) \cdot \exp(i\varphi_k)\right|^2 \qquad (3)$$

Given a set of m competing points on the screen, described by a *position vector* $\vec{q} \in \mathbb{R}^m$, the m-objective DG problem becomes:

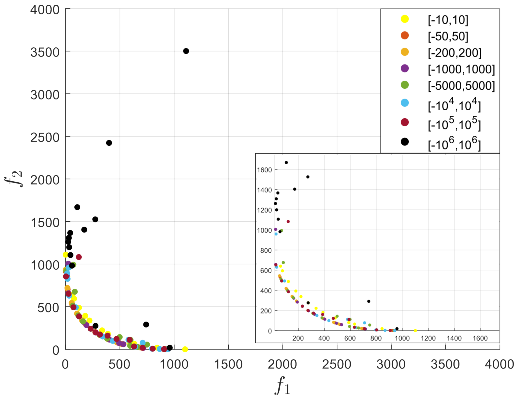
$$\vec{f}(\vec{q}, \vec{\varphi}) = \begin{pmatrix} I_{DG}(q_1, \vec{\varphi}) \\ I_{DG}(q_2, \vec{\varphi}) \\ \vdots \\ I_{DG}(q_m, \vec{\varphi}) \end{pmatrix} \longrightarrow \max \qquad (4)$$

Discrete Opportunity: Nonlinear Nonconvex Objective[6]

The Low-Autocorrelation Binary Sequence (LABS) problem is a hard combinatorial optimization problem with practical applications in electrical engineering. It is incorporated within the IOHprofiler.
Given a sequence of length n, $S := (s_1, \ldots, s_n)$ with $s_i = \pm 1$,

$$\boxed{\begin{array}{l} \textbf{[LABS]} \;\; \text{maximize } \dfrac{n^2}{2E(S)} \\ \text{subject to:} \\ \qquad E(S) := \displaystyle\sum_{k=1}^{n-1} \left(\sum_{i=1}^{n-k} s_i \cdot s_{i+k} \right)^2 \\ \qquad s_i \in \{-1, +1\} \;\; \forall i \in \{1 \ldots n\} \end{array}} \tag{5}$$

Mixed-Integer Opportunity: White-Box Solvers' Deficiency on moMIQP

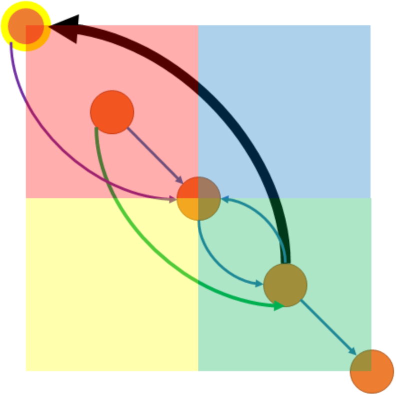


Shir and Emmerich, "Comparing White-Box to Black-Box Solvers over Multi-Objective Mixed-Integer Quadratic Models", *submitted* (2023).

Complexity of Unbounded Problems

Theorem [Jeroslow, 1973][7]
A quadratically-constrained mixed-integer unbounded problem is undecidable.

1. A generalized unbounded MIQP **cannot be linearized** (even if the integers $x_i \ : \ i \in I$ may be linearized using auxiliary binaries, the multiplication of two (loosely bounded) decision variables within $\vec{x}$ cannot be precisely linearized (rather could be piecewise approximated upon separation).
2. Despite this theoretical complexity, there has been **much practical progress in treating Quadratically-Constrained problems**[8] (e.g., by reformulation to a bilinear programming problem with integer variables, or by diverting to Mixed-Integer Second-Order Cone Programming when the model permits).



Discussion

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[3] C. Blum and G. R. Raidl, *Hybrid Metaheuristics: Powerful Tools for Optimization*. Artificial Intelligence: Foundations, Theory, and Algorithms, Switzerland: Springer International Publishing, 2016.

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[6] C. Doerr, F. Ye, N. Horesh, H. Wang, O. M. Shir, and T. Bäck, "Benchmarking discrete optimization heuristics with IOHprofiler," *Applied Soft Computing*, vol. 88, p. 106027, 2020.

[7] R. G. Jeroslow, "There cannot be any algorithm for integer programming with quadratic constraints," *Operations Research*, vol. 21, no. 1, pp. 221–224, 1973.

[8] C. Bliek, P. Bonami, and A. Lodi, "Solving Mixed-Integer Quadratic Programming problems with IBM-CPLEX: a progress report," in *Proceedings of the 26th RAMP Symposium, Hosei University, Tokyo*, 2014.