Pareto Recommendation via Graph-Based Modeling for Interactive Decision-Making

Dr. Ofer M. Shir

School of Computer Science, Tel-Hai College, Israel

ofersh@telhai.ac.il

Based on a recent paper that appeared in EVOLVE-2014, joint work with IBM-Research and the Technion

Presentation Overview

- Motivation: Consumable Pareto Analytics
- Automated Recommendation
- Gain Desire *versus* Loss Aversion
- Graph Modeling and Subset Attainment

Motivation: Consuming Optimality

- Following a multiobjective optimization process the Pareto Frontier is attained (deterministic algorithms vs. heuristics)

- Our scenario: The DM's preferences and tendencies are known (e.g., following an elicitation process)

- Typical real-world client's request –
  - ***"can you narrow-down the Frontier to recommended solutions only?"***
- Goal
  - Derive a subset of solution-points on the Frontier
  - Account for gain-prone and loss-averse subsets

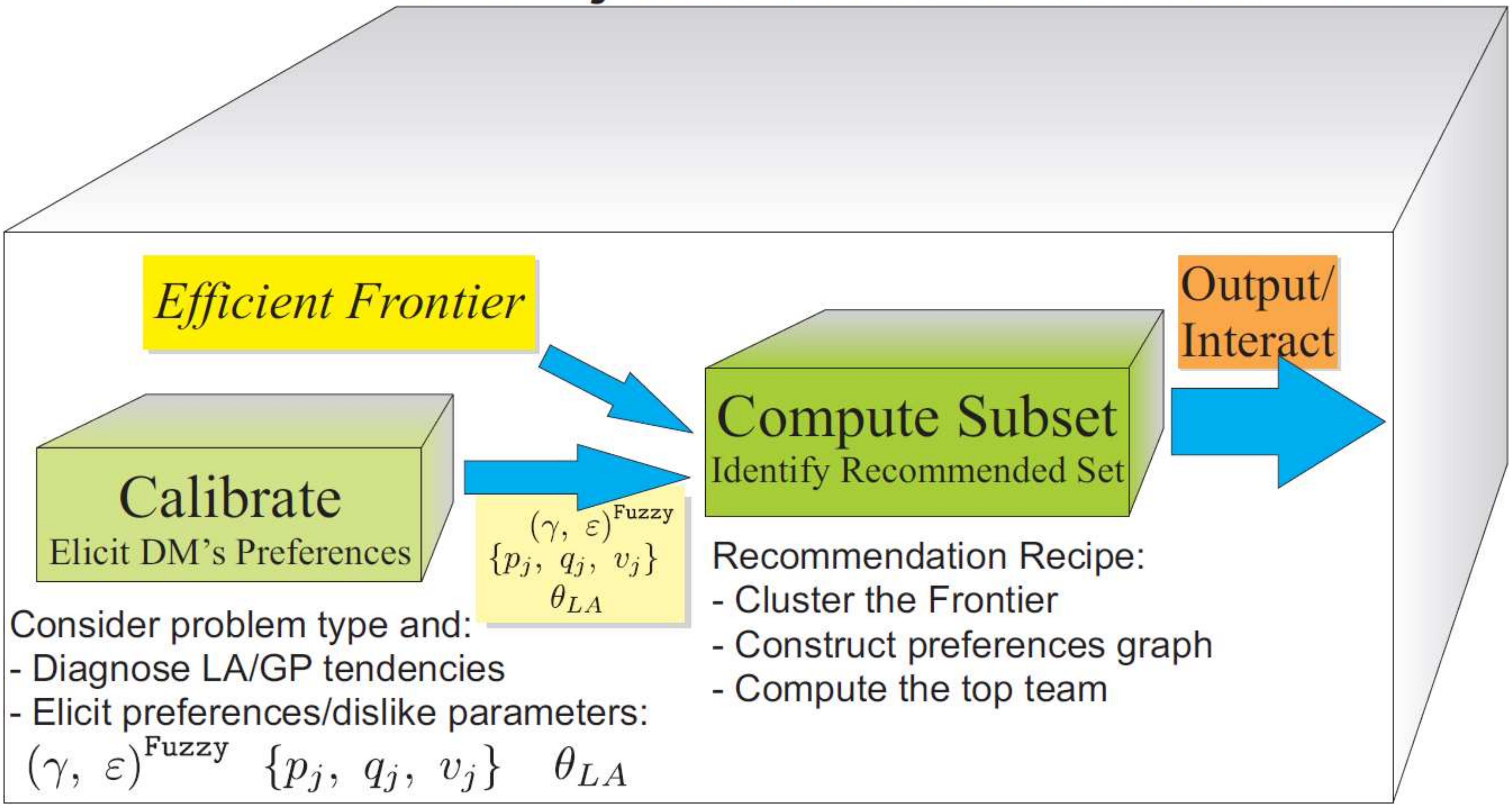
- The means: Graph-Based Modeling

Related Work

- Relevant studies lie in the domains of Multi-Criterion Decision Making and Interactive Recommender Systems
- Preference Elicitation
  - Multi-Attribute Utility Theory, Analytical Hierarchy Process
- Recommendation
  - ELECTRE

AUTOMATED RECOMMENDATION

Multiobjective Recommendation Process



Proposed Framework

recommend(Efficient Frontier $\mathcal{F}$, numClusters κ, int mode)

$\Gamma \longleftarrow$ `cluster` $(\mathcal{F},\ \kappa)$
for $i = 1 \ldots \kappa$ **do**
    $\mathcal{G}_i \longleftarrow$ `calcOutrankingGraph` $(\mathcal{F}\,(\Gamma(i,:)))$
    if `mode`==**GP then**
        $\mathcal{W}_i \longleftarrow$ `selectOffensiveTeam` $(\mathcal{G}_i)$
    else
        $\mathcal{W}_i \longleftarrow$ `selectDefensiveTeam` $(\mathcal{G}_i)$
return $\{\mathcal{W}_i\}_{i=1}^{\kappa}$ /* top teams per cluster */

Clustering

- Divide the Pareto hyper-surface into smaller regions and let the DM focus only on regions they find interesting:
- The solutions in each cluster are independent of the other clusters.
- Eventually, we select n_i winners, out of N_i solutions ($n_i/N_i \approx n/N$)

Pairwise Comparisons

- Measure for each pair of solutions the **degree of certainty** that solution ***a*** outperforms solution ***b***
  - Simpler than conducting global prioritization over a set of solutions: $$\sum_{1\leq i\leq \kappa} 2\binom{N_i}{2} \sim \frac{1}{\kappa} 2\binom{N}{2}$$
  - Seems like a natural task for the DM
- We consider three *estimation techniques*:
  1. K-Optimality feat. Fuzzy Logic
  2. ELECTRE-III
  3. ELECTRE-IS

Graph Construction

`calcOutrankingGraph`(solutions $\mathcal{F}$)

`initialize pairwise preference matrix` $\Omega = (\omega_{i,j}) \in \mathbb{R}^{|\mathcal{F}| \times |\mathcal{F}|}, \; \omega_{i,i} = 0$

$V \longleftarrow \mathcal{F}$

$E \longleftarrow \emptyset$

for $i = 1 \ldots |\mathcal{F}|$ **do**

    for $j = 1 \ldots |\mathcal{F}|$ **do**

        switch (`mode`)

        case `K-OPT`**:**

            $\omega_{i,j} \longleftarrow 1/\min_{k}\left(\boldsymbol{f}^{(i)} \prec_k \boldsymbol{f}^{(j)}\right)$

        case `ELECT-III`**:**

            $\omega_{i,j} \longleftarrow \sigma\left(\boldsymbol{f}^{(i)}, \; \boldsymbol{f}^{(j)}\right)$

        case `ELECT-IS`**:**

            $\omega_{i,j} \longleftarrow \max_s\left(\boldsymbol{f}^{(i)} \preceq_s \boldsymbol{f}^{(j)}\right)$

        end switch

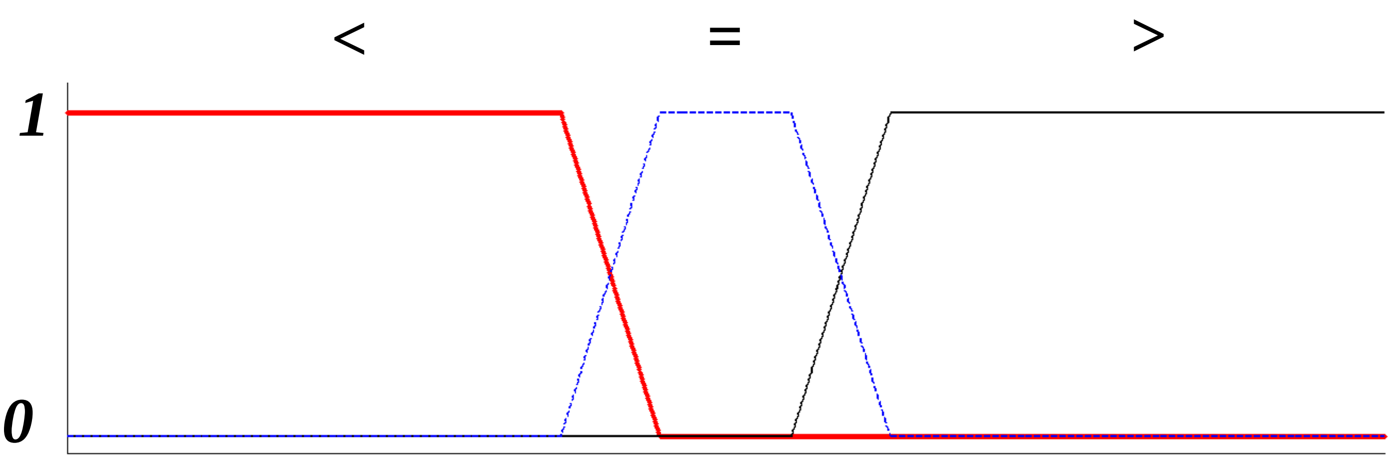
        $E \longleftarrow E \cup (i \rightsquigarrow j, \; w = \omega_{i,j})$

return $\mathcal{G} = \{V, E\}$ /* complete directed graph */

Fuzzy Logic Relations

When comparing two solutions, we account for the degree of improvement in each coordinate by means of fuzzy membership functions (***b****etter*/***e****qual*/***w****orse*) :

$$n_{\{b,e,w\}}\left(\boldsymbol{f}^{(1)},\boldsymbol{f}^{(2)}\right) \equiv \sum_{i=1}^{m} \mu^{(i)}_{\{b,e,w\}}\left(f_i^{(1)} - f_i^{(2)}\right)$$



Fuzzy Logic: *k*-dominance and *k*-optimality

- We state that solution $\boldsymbol{f_1}$ *k*-dominates $\boldsymbol{f_2}$ if and only if,

$$\left(n_e\left(\vec{f}^{(1)},\vec{f}^{(2)}\right)<m\right) \quad \text{and} \quad \left(\frac{n_w\left(\vec{f}^{(1)},\vec{f}^{(2)}\right)}{n_b\left(\vec{f}^{(1)},\vec{f}^{(2)}\right)}\right)\leq k$$

  and denote it as $\boldsymbol{f}^{(1)} \prec_k \boldsymbol{f}^{(2)}$.

➢ Note: $\varepsilon=0, \quad k=0$ will reduces this relation to Pareto-dominance relation

- As a result, we may define the following preference function:

$$\omega_{i,j} \longleftarrow 1/\min_k\left(\boldsymbol{f}^{(i)} \prec_k \boldsymbol{f}^{(j)}\right)$$

ELimination Et Choix Traduisant la REalité (ELECTRE)

- Consider the binary comprehensive outranking relation S; $a'Sa$ holds when a', with respect to every criterion, is at least as good as a

$$a'S_j a \quad \text{iff} \quad f_j(a') \le f_j(a) + q_j$$

- The subset of all criteria that are in concordance with the assertion $a'Sa$ is called **the concordant coalition** (with this assertion). It is denoted by $C(a'Sa)$

- The j[th] criterion is in discordance with the assertion $a'Sa$ if and only if $aP_j a'$

$$aP_j a' \quad \text{iff} \quad f_j(a) < f_j(a') - p_j$$

- The subset of all criteria that are in discordance with the assertion $a'Sa$ is **the discordant coalition** $C(aPa')$

- Given the set of criteria (objectives) $\boldsymbol{F}$, we may conclude

$$C(a'Sa) \cap C(aPa') = \varnothing$$

$$C(a'Sa) \cup C(aPa') \subset F$$

Concordance/Discordance, Hesitation, Veto

- There are scenarios where $C\left(a'Sa\right) \bigcup C\left(aPa'\right) \neq F$; these would hold when

$$aQ_j a' \quad \text{iff} \quad f_j\left(a\right) + p_j \geq f_j\left(a'\right) > f_j\left(a\right) + q_j \quad \left(p_j > q_j\right)$$

- Finally, with each ordered pair $\left(a', a\right)$, a partition of $\boldsymbol{F}$ into three subsets is associated:

$$C\left(a'Sa\right) \bigcup C\left(aQa'\right) \bigcup C\left(aPa'\right) = F$$

- Upon the validation of $a'Sa$, we consider discordant criteria.
- We also consider a *veto threshold*, $\boldsymbol{v_j}$, defined by means of the following statement:

$f_j\left(a\right) - f_j\left(a'\right) < -v_j$ is incompatible with the assertion $a'Sa$ whatever the other performances are.

Utilizing ELECTRE

- We consider 2 specific variants and derive estimation metric (details excluded):
  - ELECTRE-III $\omega_{i,j} \longleftarrow \sigma\left(\boldsymbol{f}^{(i)},\ \boldsymbol{f}^{(j)}\right)$

$$\sigma(a,b) = c(a,b) \cdot \prod_{j:d^{(j)}_{(p,v)}(a,b)>c_{(q)}(a,b)} \frac{1-d^{(j)}_{(p,v)}(a,b)}{1-c_{(q)}(a,b)}$$

  - ELECTRE-IS $\omega_{i,j} \longleftarrow \max_s\left(\boldsymbol{f}^{(i)} \preceq_s \boldsymbol{f}^{(j)}\right)$

  where $a \preceq_s b$ holds if and only if

$$\begin{cases} c_{(q)}(a,b) \geq s \\ \forall j \;\; d^{(j)}_{(p,v)} \geq -v + (v-p) \cdot w\left(s, c_{(q)}(a,b)\right) \end{cases}$$

Outranking Aftermath

- We construct a complete directed graph with weights:
  - Minimal value 0: $\boldsymbol{a}$ is certainly not better than $\boldsymbol{b}$
  - Large positive value: the degree to which $\boldsymbol{a}$ is preferred over $\boldsymbol{b}$
- Calibration of either methods is necessary.
- Fuzzy scoring reflects the <u>Gain-Prone POV</u>.
- ELECTRE methods reflect the <u>Loss-Averse POV</u>.

Selection: **L**oss **A**verse ***versus*** **G**ain-**P**rone

Inspired by studies of Kahneman-Tversky, we devise:

- GP track: solutions that "win the most" form the top ***offensive team***
- LA track: solutions that "lose the least" form the top ***defensive team***

Suggested Selection: Graph Kernels

- A graph kernel is the subset of vertices that is both independent and dominating:

$$\text{if} \quad u, v \in \mathrm{K} \quad \text{then} \quad (u,v),(v,u) \notin \mathrm{K}$$

$$\text{for any} \quad v \notin \mathrm{K} \quad \text{there exists}\; u \in \mathrm{K} : (u,v) \in \mathrm{K}$$

- Kernels are typically computed in ELECTRE-based selection schemes as the output of the selection process.
- We argue that kernels are inappropriate for our selection process due to the following reasons:
  - Defined for unweighted graphs
  - No control over its size; may be empty

GP Track: The Top Offensive Team

- Algorithm 1 – naïve:
  1. For each vertex: $deg(v) = \max_{e \in \delta_{out}(v)} w_e$
  2. Select top n_i vertices

- Algorithm 2 – a relaxation of the *Dominating Set Problem* for weighted graphs:
  1. For each set D, we define the covering degree of each vertex as,

$$cvr\left(\mathcal{D}, v\right) = \begin{cases} 1 & \text{if } v \in \mathcal{D} \\ \max\limits_{u \in \mathcal{D}, (u,v) \in E} w(u, v) & \text{otherwise} \end{cases}$$

  2. We define the *covering degree* of each set as the total degree of all vertices,

$$cvr\left(\mathcal{D}\right) = \sum_{v \in V} cvr\left(\mathcal{D}, v\right)$$

  3. Solve: $\max\limits_{|\mathcal{D}| \leq n} cvr\left(\mathcal{D}\right)$

Solving $\max_{|\mathcal{D}| \leq n} cvr(\mathcal{D})$

1. Greedy: $\mathcal{D}^{k+1} \leftarrow \mathcal{D}^k \cup \left\{ \arg\max_{v \in V} \left\{ cvr\left(\mathcal{D}^k \cup \{v\}\right) - cvr\left(\mathcal{D}^k\right) \right\} \right\}$
   Since *cvr(D)* constitutes a *submodular monotone function*, this approach guarantees a (*1-1/e*)-approximation to it !

2. Mixed-Integer LP (MILP; employing ILOG-CPLEX):

Binary decision variables: $\texttt{sol}[1..|V|]$ $\texttt{cover}[1..|V|][1..|V|]$

where $\texttt{cover}[v][u] = 1$ translates to vertex *v* to cover vertex *u*

$$\textbf{[P1]} \quad \text{maximize} \sum_{v \in V} \sum_{u \in V} w(u,v) \cdot \texttt{cover}[v][u]$$

subject to:

$$\sum_{v \in V} \texttt{sol}[v] \leq N_{\text{rec}}$$

$$\sum_{v \in V} \texttt{cover}[v][u] \leq 1 \ \forall u \in V$$

$$\texttt{cover}[v][u] \leq \texttt{sol}[v] \ \forall u \in V \ \forall v \in V$$

LA Track: The Top Defensive Team

- Algorithm 3 – naïve:

1. For each vertex: $deg(v) = \max_{e \in \delta_{in}(v)} w_e$
2. Select tail n_i vertices

- Algorithm 4 :

1. For each resisting set R, we define the degree of each vertex as,

$$deg\left(\mathcal{R}, u\right) = \sum_{v \in \mathcal{R} \setminus u} w\left(u, v\right)$$

2. The resistance degree of each set is then defined as the maximal degree of all vertices (i.e., the strongest offence on R):,

$$res\left(\mathcal{R}\right) = \max_{u \in V} deg\left(\mathcal{R}, u\right)$$

3. Solve: $\min_{|\mathcal{R}| \geq n} res\left(\mathcal{R}\right)$

Solving $\min_{|\mathcal{R}| \geq n} res(\mathcal{R})$

1. Greedy: $\mathcal{R}^{k+1} \longleftarrow \mathcal{R}^k \cup \left\{ \arg\min_v \sum_{\mathcal{R}^k \cup \{v\}} w(u,v) \right\}$

2. MILP:

Binary decision variables: $\texttt{sol}[1..|V|]$

$$\textbf{[P2]} \quad \text{minimize } \texttt{t}$$

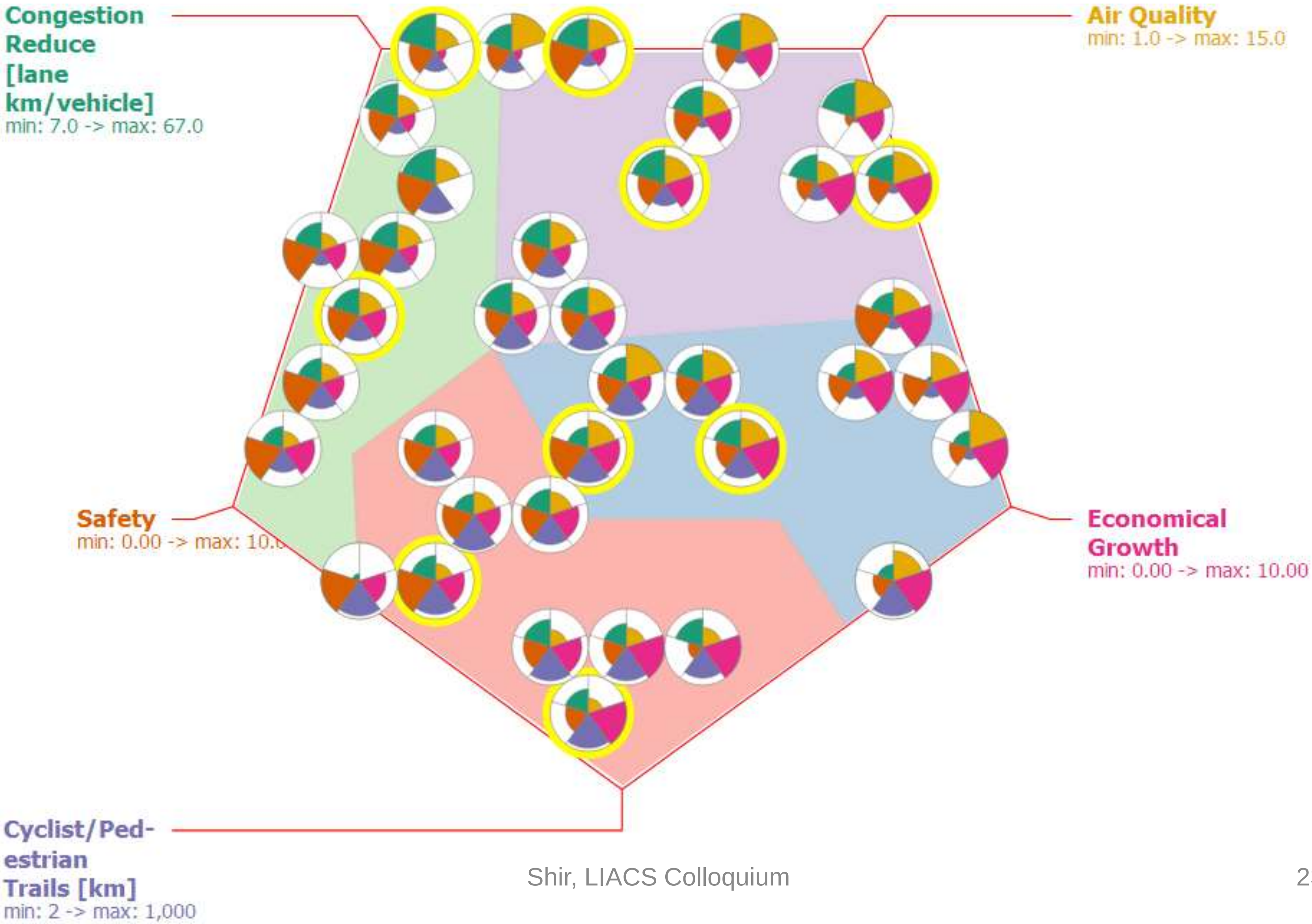
subject to:

$$\sum_{v \in V} \texttt{sol}[v] \geq N_{\text{rec}}$$

$$\sum_{v \in V} w(u,v) \cdot \texttt{sol}[v] \leq \texttt{t} \ \forall u \in V$$

Demonstration: 5-Objective Problem

Visualization by means of SOMMOS:



Discussion

- ELECTRE is oriented toward loss aversion, but does not excel in distinguishing between domination to quasi-domination
- On the other hand, the Fuzzy K-Domination approach does not differentiate between loss aversion to gain favoring
- We also propose a hybrid approach –
  - As long as we are not imposed to a significant loss (ELECTRE-wise), we would like to rank according to gains/winnings
  - We utilize ELECTRE to evaluate incredibility and Fuzzy/K-Optimality to measure preference

$$Hyb(a,b) = \begin{cases} 0 & \text{if } \sigma < thresh \\ F(a,b) & \text{otherwise} \end{cases} \qquad F(a,b) = \min\left\{0, \frac{n_b - n_w}{M}\right\}$$

- LA is not GP-dual!

References

- Shir, O.M., Chen, Sh., Amid, D., Margalit, O., Masin, M., Anaby-Tavor, A., Boaz, D.: Pareto Landscapes Analyses via Graph-Based Modeling for Interactive Decision-Making. Advances in Intelligent Systems and Computing volume 288 (EVOLVE-2014), Springer (2014) 97–113
- Chen, Sh., Amid, D., Shir, O.M., Boaz, D., Schreck, T., Limonad, L.: Self-Organizing Maps for Multi-Objective Pareto Frontiers. In: Proceedings of the Pacific Visualization Symposium, PacificVis-2013, IEEE (2013) 153–160
- Roy, B.: The Outranking Approach and the Foundations of ELECTRE Methods. Theory and Decision 31 (1991) 49–73
- Saaty, T.L.: The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation. McGraw-Hill (1980)
- Kahneman, D., Tversky, A.: Prospect Theory: An Analysis of Decision Under Risk. Econometrica 47(2) (March 1979) 263–291
- Farina, M., Amato, P.: Fuzzy Optimality and Evolutionary Multiobjective Optimization. In: Evolutionary Multi-Criterion Optimization. Volume 2632 of Lecture Notes in Computer Science. Springer Berlin Heidelberg (2003) 58–72
- Nemhauser, G.L., Wolsey, L.A., Fisher, M.L.: An Analysis of Approximations for Maximizing Submodular Set Functions - I. Mathematical Programming 14(1) (1978) 265–294